Sensitivity of the nuclear equation of state towards relativistic effects [1]

Rajeev K. Puri, E. Lehmann, Amand Faessler and S.W. Huang

Institut für Theoretische Physik der Universität Tübingen,
Auf der Morgenstelle 14, D72076, Tübingen, Germany.

February 4, 2008 **Abstract:**

We analyze relativistic effects in transverse momentum using Quantum Molecular Dynamics [QMD] and its covariant extension Relativistic Quantum Molecular Dynamics [RQMD]. The strength of the relativistic effects is found to increase with the bombarding energy and with an averaged impact parameter. The variation in the intensity of the relativistic effects with variation in the mass of the colliding nuclei is not systematic. Furthermore, the hard EOS is affected drastically by the relativistic effects whereas the soft EOS is affected less. Our analysis shows that up to the bombarding energy of 1 GeV/nucl., the influence of relativistic effects is small. Whereas at higher energies, relativistic effects become naturally very important.

In last years, it has become clear that for analyzing the heavy ion collisions at intermediate energies, one needs a Lorentz-invariant theory [1-6]. Therefore, it is important to study the consequences of the covariant treatment of the dynamics on observables calculated with different nuclear **E**quations **O**f **S**tate [EOS]. One of the most promising quantities which are sensitive to different EOS's is the transverse momentum [also called "flow" in the literature]. Moreover, due to the fact that the transverse momentum is affected drastically by the model parameters, by the bombarding energy, by the masses of the colliding nuclei and by the impact parameters [7], [8], it is important to study the relativistic effects on the transverse momentum using different bombarding energies, masses of the colliding nuclei and impact parameters.

The differences of observables obtained between a relativistic and a non-relativistic treatment have different physical origins. Some of them are covariant treatment of the dynamics, relativistic forces, retardation effects or meson radiations [1-6]. Relativistic effects which we are going to discuss are the one arise from full covariant treatment of the dynamics [RQMD] [2] compared to QMD results. To keep the influence of the relativistic mean field away from this analysis, we do not use the relativistic mean field. Rather we generalize the normal Skyrme force in such a way that it gains well defined Lorentz properties, e.g. is treated as a Lorentz-scalar.

To analyze the transverse momentum, we use **Q**uantum **M**olecular **D**ynamics [QMD] and its covariant extension **R**elativistic **Q**uantum **M**olecular **D**ynamics [RQMD]. The RQMD approach was developed by the Frankfurt group [1] and is massively used nowadays at ultra-relativistic energies. We use here a similar method in order to study relativistic effects at intermediate energies. The details of QMD and RQMD can be found in refs.[8] and [2,3], respectively.

The QMD model is a semi-classical model where important quantum features like stochas-

tic scattering, Pauli-principle, creation and reabsorption of the resonances, particle production etc. are also considered. The particles are propagated using the classical Hamiltonian. During the propagation particles can also collide elastically or inelastically [8]. The RQMD approach contains a multi-time description and propagates the particles in a covariant fashion. This covariant propagation is coupled with the above mentioned quantum features [2,3]. The main notable differences between QMD and RQMD are:

1. In RQMD, we have initially a Lorentz-contracted distribution in coordinate space and an elongated distribution in momentum space.

2. The RQMD formalism contains a multi-time description which means that each particle carries its own time coordinate. These different time coordinates of baryons can play an important role in the time evolution of the heavy ion collisions. [2].

3. The RQMD approach contains a full covariant treatment of the collision part and the Pauli-blocking. Here in the present study, we include in RQMD and QMD $\Delta(1232)$ and $N^*(1440)$ resonances . In the energy region we are interested, these resonances are the main dominating resonances.

4. In RQMD, the mean field is a Lorentz-scalar whereas in QMD it is a zero component of the Lorentz vector. Due to the covariant feature, the interactions in RQMD are defined as a function of the distance between the particles in their centre-of-mass system. Therefore, in a moving frame these interactions are not spherical but are Lorentz contracted in the direction of the motion of the two interacting particles. Hence, the strength of the interaction depends strongly on the direction of the center-of-mass motion of the two nucleons in the rest frame of the two nuclei. When the initial coordinate space distribution is Lorentz contracted then, naturally, the density of a fast moving nucleus becomes much larger than the normal one. If one implements this feature in normal QMD then one finds that it can lead to a tremendous enhancement in the transverse flow. When one uses this feature in covariant

3

RQMD, this artificial repulsion due to the initial contraction of the coordinate space is partially counterbalanced and thus the quantities like density or flow in RQMD lies between normal QMD and QMD with a contracted initial coordinate space distribution [2].

To study the transverse flow, one oftenly takes the average-in-plane transverse momentum which is defined as:

$$< P_x^{dir} > = \frac{1}{N} \sum_{i=1}^{N} \text{sign}[Y_i] \ P_i^x, \tag{1}$$

where Y_i and P_i^x are the rapidity and transverse momentum of the i-th particle, respectively.

The evolution of $< P_x^{dir} >$ flow has been studied and discussed extensivly in past several years [7-8]. Here our interest is to study the influence of a covariant formalism compared to a non-covariant formalism. Therefore, instead of looking to the absolute values of the flow, we construct another quantity which can give us a kind of measure of the **S**ize **O**f **R**elativistic **E**ffects [SORE] :

$$P_x^{SORE} = \left| \frac{< P_x^{dir} > [RQMD] - < P_x^{dir} > [QMD]}{< P_x^{dir} > [QMD]} \right|. \tag{2}$$

The main advantage of P_x^{SORE} is that it gives us a direct measure of either the enhancement or the reduction in the flow using RQMD with respect to the QMD. To treat the enhancement and the reduction at equal footing, we take the absolute value of P_x^{SORE}.

In case of a covariant formalism, the important factors are the Lorentz properties which contain the factor γ [$= \frac{1}{\sqrt{(1-\beta^2)}}$, where β is the velocity of the particle as compared to the velocity of light] and the different time coordinates of the baryons. Therefore, in fig. 1, we study the P_x^{SORE} (obtained at the final stage of the reaction) as a function of the bombarding energy E_{lab}. It is impressive to note that at 50 MeV/nucl., RQMD and QMD show good agreement. The relativistic effects increase as expected with increasing bombarding energy. This is true for both the hard and the soft EOS's. One also sees that up to 1 GeV/nucl., the intensity of these relativistic effects is very small. As the flow is a very sensitive quantity,

4

one has to consider relativistic effects only if $< P_x^{SORE} >$ lies above 20%. Therefore, one can use non-covariant models up to 1 GeV/nucl. safely. But at higher energies one sees clearly strong relativistic effects in the transverse momentum. We also note that these effects do not grow in a simple systematic way as a function of γ_{cm}^{rel} [$= \gamma_{cm} - \gamma_{cm}(\beta = 0)$] or β_{cm}. Thus the dynamical relativistic effects are not only due to compression which is created by the initial Lorentz contraction of the coordinate space distribution.

To study the relativistic effects as a function of the impact parameter, we choose the bombarding energy of 1.5 GeV/nucl.. At this energy one has medium size relativistic effects (see fig.1). In fig. 2 we plot P_x^{SORE} as a function of the b_{scale}[$= b/(R_T + R_P)$, where R_T and R_P are the radius of the target and projectile, respectively]. It is interesting to see that both soft and hard EOS's show a similar behaviour i.e. the size of relativistic effects is less for the central collisions. After reaching a maximum value, the relativistic effects start to decrease with increasing impact parameter. The main notable results are : (1) The maximum relativistic effects for the hard EOS are about 170% whereas for the soft EOS, they are less than 90%. (2) The value of the P_x^{SORE} as a function of the impact parameter varies between 26 % to 170 %. This clearly stresses that in order to establish some effects, one has to take care of the whole impact parameter range.

To confront the calculations with experimental data, one has to average over a certain impact parameter range. One often averages by giving equal weights to all impact parameters (like in the case of the radial flow analysis of the FOPI collaboration [9]).

Therefore to establish an average size of relativistic effects, we do a similiar averaging. Hence, we define an averaged $< P_x^{SORE} >$ as

$$< P_x^{SORE} >= \left| \frac{\sum_{b_{scale}} < P_x^{dir} > [RQMD] - \sum_{b_{scale}} < P_x^{dir} > [QMD]}{\sum_{b_{scale}} < P_x^{dir} > [QMD]} \right|. \qquad (3)$$

This averaged quantity $< P_x^{SORE} >$ gives us the size of the relativistic effects averaged

over an impact parameter range chosen. We averaged differently : (i) First we averaged over so called **C**ollision **D**ominated impact parameters i.e. $0 < b_{scale} < 0.5$ (defined as $< P_x^{SORE} >_{CD}$). (ii) Second we averaged over the impact parameter range $0.5 \leq b_{scale} \leq 1.0$. This region is dominated by the **M**utual **I**nteractions [self-consistent field, the number of collisions is small]. We label this as $< P_x^{SORE} >_{MID}$. From fig. 2, one can see that the size of $< P_x^{SORE} >_{CD}$ (dotted lines) is about 49% for the hard EOS and about 36 % for the soft EOS. The mutual interactions dominated average [$< P_x^{SORE} >_{MID}$] shows very large effect (dashed lines) [i.e. 135.8% effect using hard EOS and 85.6% using the soft EOS]. This can be understood by the fact that in case of central and semi-central collisions, the self consistent mean field does not play an important role. Whereas for peripheral collisions, the importance of the covariant treatment of the dynamics increases. The interesting point is that when one averages over the full impact parameter range [i.e. $0.0 < b_{scale} \leq 1.0$], one finds that the hard EOS shows about a 90% effect whereas the soft EOS shows a less than 50 %. This is remarkable. The hard and soft EOS's show a very different sensitivity towards the covariant treatment of the dynamics. This also indicates that a full covariant treatment gets larger differences in the transverse flow using the hard and the soft EOS's which in a non-covariant model are always small. This different response of different EOS's towards relativistic effects stresses that one cannot estimate the size of relativistic effects by rescaling the results obtained in a non-covariant approach.

One of the important features of the Constraint Hamiltonian Dynamics is that it contains a multi-time description. In this formalism, each particle carries its own time co-ordinate in a certain frame of reference. The maximum time difference in the time coordinates of particles at a fixed bombarding energy is roughly proportional to the radius of the nucleus. Thus the ratio of the maximum time difference to the radii of the nuclei at a given bombarding energy should be roughly the same for all masses. Therefore, the study of the relativistic effects

using different colliding nuclei will give us the possibility to look whether relativistic effects are due to different time coordinates only. If one gets similar effects for all masses then, naturally, the different time coordinates of particles will be the only cause for the relativistic effects. Therefore, in figure 3, we plot the P_x^{SORE} as a function of the masses of the colliding nuclei. Note that here the bombarding energy (and hence the γ) is the same for all mass systems. Due to fact that RQMD simulations take extensive computer time, we have to restrict ourselves for masses A = 80 only. The statistical error in these calculations is about 2% for light systems (like C-C) and about 4% for the heaviest system (Ca-Ca). One clearly sees that the size of the relativistic effects differs appreciably. This shows that the relativistic effects in the present study are not only created by the multi-time description, but they can be due to a number of other causes which have been discussed above. Nevertheless the different time coordinates of the baryons play an important role in heavy ion collisions.

Concluding, we have analyzed the strength of the relativistic effects in the transverse flow by studying a variety of reactions using RQMD and QMD. The relativistic effects which are originating from the covariant treatment of the formalism are found to grow with the bombarding energy and also with the impact parameter. But we do not see any monotonic dependence of the relativistic effects on the masses of the colliding nuclei. These results clearly show that the relativistic effects are not only due to the compression by Lorentz contaction or due to the different time coordinates of the particles but, there are a number of physical causes which can produce these effects. An other important conclusion is that the hard and the soft EOS's show different sensitivities towards relativistic effects. One finds in a covariant calculation far more differences in the transverse flow using the the hard and the soft EOS's than in the non-covariant QMD.

References

[1] Sorge H, Stöcker H and Greiner W 1989 *Anns. of Phys.* **192** 266
    Sorge H, Stöcker H and Greiner W 1989 *Nucl. Phys.* A **498** 567c

[2] Lehmann E, Puri R K, Faessler A, Batko G and Huang S W *Phys. Rev.* C- submitted

[3] Fuchs C, Sehn L and Wolter H H 1993 *Prog. Part. Nucl. Phys.* **39** 247

[4] Ko C M, Li Q and Wang R 1987 *Phys. Rev. Lett.* **59** 1084

[5] Blättel B, Koch V, Cassing W and Mosel U 1988 *Phys. Rev.* C **38** 1767

[6] Schmidt R, Kämpfer B, Feldmeier H and Knospe O 1989 *Phys. Lett.* B **229** 197

[7] Molitoris J J, Stöcker H and Winer B L 1987 *Phys. Rev.* C **36** 220

[8] Aichelin J 1991 *Phys. Rep.* **202** 233

[9] Reisdorf W and Gobbi A, private communication for FOPI Collaboration

Figure Captions

Fig.1 The size of the relativistic effects $P_x^{SORE}\%$ (eq. 2) as a function of the bombarding energy. The reaction under consideration is ^{40}Ca-^{40}Ca at an impact parameter 2 fm. The variation of the $\gamma_{cm}^{rel}\%$ [i.e. γ_{cm}^{rel} x 100] and $\beta_{cm}\%$ [β_{cm} x100] as a function of the bombarding energy is also shown. Here γ_{cm}^{rel} is defined as $\gamma_{cm} - 1$.

Fig.2 The $P_x^{SORE}\%$ as a function of the scaled impact parameter b_{scale}. For the definition of the $< P_x^{SORE} >_{CD}$, $< P_x^{SORE} >_{MID}$, see text.

Fig.3. The same as in fig. 1 but $P_x^{SORE}\%$ is plotted as a function of mass of the colliding nuclei. Here the impact parameter is $b_{scale} = 0.25$ b_{max} [$b_{max} = R_T + R_P$] and the bombarding energy 1.5 GeV/nucl..

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9502009v1

This figure "fig2-1.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9502009v1

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9502009v1